

# Spin alignment of vector meson in $e^+e^-$ annihilation at $Z^0$ pole

Xu Qing-hua, Liu Chun-xiu and Liang Zuo-tang

*Department of Physics, Shandong University, Jinan, Shandong 250100, China*

## Abstract

We calculate the spin density matrix of the vector meson produced in  $e^+e^-$  annihilation at  $Z^0$  pole. We show that the data imply a significant polarization for the antiquark which is created in the fragmentation process of the polarized initial quark and combines with the fragmenting quark to form the vector meson. The direction of polarization is opposite to that of the fragmenting quark and the magnitude is of the order of 0.5. A qualitative explanation of this result based on the LUND string fragmentation model is given.

Spin effects in high energy fragmentation processes have attracted much attention [1-11] recently since they provide not only useful information on the spin dependence of hadronic interaction but also a promising tool to study the spin structure of nucleon. One of the important issue in this connection is the spin transfer of the fragmenting quark to the produced hadron which contains this quark. There are two related questions here, one is whether the fragmenting quark retains its polarization, the other is how one should connect the spin of the fragmenting quark to that of the hadron which contains the quark.

A series of papers [1-3, 6-11], both experimental and theoretical, have been published on this topic. These papers all concentrate on the hyperon polarization in lepton induced reactions since hyperon polarization can easily be obtained in experiments by measuring the angular distributions of its decay products. Although the available data are still far from accurate and enormous enough to make a definite conclusion, the comparison of the data and theoretical results seems to suggest that<sup>8,11</sup> the polarization of the fragmenting quark is retained in the fragmentation and that the simple SU(6) wave function can be used in connecting the spin of the fragmenting quark to that of the produced hadron which contains the quark.

It is also interesting to note that information on polarization of vector meson can also be obtained from angular distributions of their decay products. Such measurements have also been carried out by OPAL, DELPHI and ALEPH collaboration at LEP for different vector mesons in  $e^+e^-$  annihilation at  $Z^0$  pole [12-16]. The results show clearly that the produced vector meson favor the helicity zero state which implies a nonzero polarization of the vector meson in the direction perpendicular to the moving direction. There are also measurements on the off-diagonal elements of the spin density matrix. It is then natural to ask whether the data can also be understood from the same starting points as those for hyperon polarization and what they imply for the polarization of the antiquark produced in the fragmentation process of a polarized quark.

In this note, we study these questions by calculating the spin density matrix for the produced vector mesons by adding the spin of the fragmenting quark (or antiquark) with

that of the antiquark (or quark) created in the fragmentation together. With the aid of an event generator JETSET, we calculate the  $z$  dependence of the polarization by taking all different contributions into account. Here  $z \equiv 2p_V/\sqrt{s}$ , where  $p_V$  is the momentum of the vector meson,  $\sqrt{s}$  is the total  $e^+e^-$  center of mass energy. We now first outline the calculation method in the following.

Similar to that for hyperon polarization<sup>11</sup>, to calculate the polarization of vector mesons in the inclusive process  $e^+e^- \rightarrow Z^0 \rightarrow q_f^0 \bar{q}_f^0 \rightarrow V+X$ , we need to divide the produced vector mesons into the following two groups and consider them separately. (Here the subscript  $f$  denotes the flavor of the quark.) They are (a) those which contain the initial quark  $q_f^0$ 's or the initial antiquark  $\bar{q}_f^0$ 's; (b) those which don't contain the initial quark or antiquark. The spin density matrix  $\rho^V(z, k_\perp)$  for the vector meson  $V$  should be given by:

$$\rho^V(z, k_\perp) = \sum_f \frac{\langle n(z, k_\perp | a, f) \rangle}{\langle n(z, k_\perp) \rangle} \rho^V(z, k_\perp | a, f) + \frac{\langle n(z, k_\perp | b) \rangle}{\langle n(z, k_\perp) \rangle} \rho^V(z, k_\perp | b), \quad (1)$$

where  $\langle n(z, k_\perp | a, f) \rangle$  and  $\rho^V(z, k_\perp | a, f)$  are the average number and spin density matrix of vector mesons from (a);  $\langle n(z, k_\perp | b) \rangle$  and  $\rho^V(z, k_\perp | b)$  are those from (b).  $\langle n(z, k_\perp) \rangle = \sum_f \langle n(z, k_\perp | a, f) \rangle + \langle n(z, k_\perp | b) \rangle$  is the total number of vector mesons and  $k_\perp$  is the transverse momentum of the vector meson with respect to the moving direction of initial quark or antiquark. Here, in contrast to the case for  $\Lambda$  hyperon production, contributions from the decay of heavier hadrons are very small. We just treat them in the same way as those from (b).

Similar to the case for hyperon production, there are many different possibilities to produce the vector mesons in group (b) and the polarization in each possibility can be different from that in the other. Hence, it is very likely that these vector mesons as a whole are unpolarized. We will, just as we did in Ref.<sup>11</sup> for hyperons, take them as unpolarized. This means that we simply take  $\rho^V(z, k_\perp | b)$  as a unit matrix. The spin density matrix  $\rho^V(z, k_\perp | a, f)$  of the vector meson  $V$  which contains the fragmenting quark  $q_f^0$  (or antiquark  $\bar{q}_f^0$ ) and an antiquark  $\bar{q}$  (or a quark  $q$ ) which is created in the fragmentation process can be calculated from the direct product of the spin density matrix  $\rho^{q_f^0}$  for  $q_f^0$  (or  $\rho^{\bar{q}_f^0}$  for  $\bar{q}_f^0$ ) and

that  $\rho^{\bar{q}}$  for the antiquark  $\bar{q}$  (or  $\rho^q$  for  $q$ ). Now, we are going to formulate the calculation of  $\rho^V(z, k_\perp | a, f)$ . For explicity, we will take the case  $V = (q_f^0 \bar{q})$  as an example.

According to the Standard Model for electro-weak interaction, the initial quark or antiquark produced in  $e^+e^-$  annihilation at  $Z^0$  pole is longitudinally polarized and the average value of polarization is  $P_f = -0.94$  for  $f = d, s$  and  $b$ ;  $P_f = -0.67$  for  $f = u, c$ <sup>3,11</sup>. Hence the normalized spin density matrix  $\rho^{q_f^0}$  is given by

$$\rho^{q_f^0} = \begin{pmatrix} c_{1f} & 0 \\ 0 & c_{2f} \end{pmatrix}, \quad (2)$$

where  $c_{1f} = (1 + P_f)/2$ ,  $c_{2f} = (1 - P_f)/2$ . Here we use the helicity frame of  $q_f^0$ , which we denote by  $xyz$ , where  $z$ -axis is taken as the moving direction of  $q_f^0$  in the overall center-of-mass frame and  $x$ -axis as the direction of the transverse momentum of the antiquark  $\bar{q}$  with respect to the  $z$ -axis. The  $\rho^{\bar{q}}$  is taken as the most general form in the frame  $xyz$ , i.e.,

$$\rho^{\bar{q}} = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}, \quad (3)$$

where  $\vec{P}(P_x, P_y, P_z)$  is the polarization vector of  $\bar{q}$ . The spin density matrix  $\rho^{q_f^0 \bar{q}}$  of the  $q_f^0 \bar{q}$  system is obtained from Eq. (2) and Eq. (3), i.e,

$$\rho^{q_f^0 \bar{q}} = \rho^{q_f^0} \otimes \rho^{\bar{q}} = \begin{pmatrix} c_{1f} \rho^{\bar{q}} & 0 \\ 0 & c_{2f} \rho^{\bar{q}} \end{pmatrix}. \quad (4)$$

We note that the  $\rho^{q_f^0 \bar{q}}$  obtained in this way is in the basis of  $|s^{q_f^0}, s_z^{q_f^0}; s^{\bar{q}}, s_z^{\bar{q}}\rangle$ , where  $s^{q_f^0}$  and  $s^{\bar{q}}$  are the spins of  $q_f^0$  and  $\bar{q}$ , and  $s_z^{q_f^0}$  and  $s_z^{\bar{q}}$  are their  $z$  components. To obtain the spin density matrix for the meson, we need to transform it to the coupled basis  $|s, s_z\rangle$ , where  $\vec{s} = \vec{s}^{q_f^0} + \vec{s}^{\bar{q}}$ . The bases of these two representations are related to each other by a unitary matrix  $U$ ,

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}. \quad (5)$$

The spin density matrix  $\rho^m$  in the coupled basis is obtained from  $\rho^m = U\rho^{q_f^0\bar{q}}U^{-1}$ , i.e.,

$$\rho^m = \frac{1}{2} \begin{pmatrix} c_{1f}(1+P_z) & \frac{c_{1f}}{\sqrt{2}}(P_x - iP_y) & 0 & \frac{c_{1f}}{\sqrt{2}}(P_x - iP_y) \\ \frac{c_{1f}}{\sqrt{2}}(P_x + iP_y) & \frac{1}{2}(1 - P_f P_z) & \frac{c_{2f}}{\sqrt{2}}(P_x - iP_y) & \frac{1}{2}(P_f - P_z) \\ 0 & \frac{c_{2f}}{\sqrt{2}}(P_x + iP_y) & c_{2f}(1 - P_z) & -\frac{c_{2f}}{\sqrt{2}}(P_x + iP_y) \\ \frac{c_{1f}}{\sqrt{2}}(P_x + iP_y) & \frac{1}{2}(P_f - P_z) & -\frac{c_{2f}}{\sqrt{2}}(P_x - iP_y) & \frac{1}{2}(1 - P_f P_z) \end{pmatrix}. \quad (6)$$

Hence the spin density matrix  $\rho'^V(a, f)$  for the vector meson V in the basis of  $|s^V, s_z^V\rangle$  can be read out from Eq. (6) as follows:

$$\rho'^V(a, f) = \frac{2}{3 + P_f P_z} \begin{pmatrix} c_{1f}(1+P_z) & \frac{c_{1f}}{\sqrt{2}}(P_x - iP_y) & 0 \\ \frac{c_{1f}}{\sqrt{2}}(P_x + iP_y) & \frac{1}{2}(1 - P_f P_z) & \frac{c_{2f}}{\sqrt{2}}(P_x - iP_y) \\ 0 & \frac{c_{2f}}{\sqrt{2}}(P_x + iP_y) & c_{2f}(1 - P_z) \end{pmatrix}. \quad (7)$$

In addition, we obtain also the ratio  $P/V$  of pseudoscalar meson to vector meson from Eq. (6) as

$$P/V = \frac{1 - P_f P_z}{3 + P_f P_z}. \quad (8)$$

We can see that  $P/V = 1/3$  at  $P_z = 0$  and  $P/V > 1/3$  at  $P_f P_z < 0$ .

To compare  $\rho'^V(a, f)$  with the data [12-16], we need to transform it to the helicity basis of the vector meson, i.e., the helicity beam frame which we denote by  $OXYZ$ . In this frame,  $Z$ -axis is taken as the moving direction of the vector meson and  $Y$ -axis is taken as the vector products of  $Z$  and the  $e^-$  beam direction (see Fig. 1). This frame transformation can be carried out by two successive rotations: first a rotation of angle  $\beta$  around  $y$  axis which transforms  $oxyz$  to  $ox'yZ$ , then a rotation of angle  $\gamma$  around the  $Z$ -axis which transforms  $ox'yZ$  to  $OXYZ$ , i.e.,

$$\rho^V(z, k_\perp | a, f) = D^\dagger(\beta, \gamma) \rho'^V(a, f) D(\beta, \gamma), \quad (9)$$

where  $D(\beta, \gamma)$  is the rotation matrix

$$D(\beta, \gamma) = \begin{pmatrix} e^{-i\gamma} \cos^2 \frac{\beta}{2} & -\frac{1}{\sqrt{2}} \sin \beta & e^{i\gamma} \sin^2 \frac{\beta}{2} \\ \frac{1}{\sqrt{2}} e^{-i\gamma} \sin \beta & \cos \beta & -\frac{1}{\sqrt{2}} e^{i\gamma} \sin \beta \\ e^{-i\gamma} \sin^2 \frac{\beta}{2} & \frac{1}{\sqrt{2}} \sin \beta & e^{i\gamma} \cos^2 \frac{\beta}{2} \end{pmatrix}. \quad (10)$$

Here  $\beta$  is the angle between the momentum of the initial quark  $q_f^0$  and that of the vector meson and  $\gamma$  is the angle between the  $y$ -axis and  $Y$ -axis (see Fig. 1). Clearly,  $\beta$ ,  $\gamma$ ,  $z$  and  $k_\perp$  are related to each other by  $\sin \beta = 2k_\perp/(z\sqrt{s})$  and

$$\cos \gamma = \frac{k_z \cos \phi \sin \theta - k_\perp \cos \theta}{\sqrt{k_\perp^2 \sin^2 \phi + (k_z \sin \theta - k_\perp \cos \phi \cos \theta)^2}}, \quad (11)$$

where  $k_z = \sqrt{z^2 s/4 - k_\perp^2}$  and  $\sin \gamma < 0$  for  $\phi < \pi$  and  $\sin \gamma > 0$  for  $\phi > \pi$ ;  $\theta$  is the angle between the moving direction of the initial quark and the  $e^-$  beam direction in the overall center-of-mass frame and  $\phi$  is the azimuthal angle of  $k_\perp$  with respect to the plane of the initial quark and  $e^-$  beam.

After some straightforward algebra, we obtain the following expressions for the matrix elements which have been measured experimentally.

$$\rho_{00}^V(z, k_\perp | a, f) = \frac{1}{3 + P_f P_z} [1 - P_f (P_z \cos 2\beta + P_x \sin 2\beta)], \quad (12)$$

$$\text{Re} \rho_{1,-1}^V(z, k_\perp | a, f) = \frac{P_f \sin \beta}{3 + P_f P_z} [\cos 2\gamma (P_z \sin \beta - P_x \cos \beta) - P_y \sin 2\gamma], \quad (13)$$

$$\text{Im} \rho_{1,-1}^V(z, k_\perp | a, f) = \frac{P_f \sin \beta}{3 + P_f P_z} [\sin 2\gamma (P_z \sin \beta - P_x \cos \beta) + P_y \cos 2\gamma], \quad (14)$$

$$\begin{aligned} & \text{Re} [\rho_{1,0}^V(z, k_\perp | a, f) - \rho_{0,-1}^V(z, k_\perp | a, f)] \\ &= \frac{\sqrt{2} P_f}{3 + P_f P_z} [\cos \gamma (P_x \cos 2\beta - P_z \sin 2\beta) + P_y \cos \beta \sin \gamma], \end{aligned} \quad (15)$$

$$\begin{aligned} & \text{Im} [\rho_{1,0}^V(z, k_\perp | a, f) - \rho_{0,-1}^V(z, k_\perp | a, f)] \\ &= \frac{\sqrt{2} P_f}{3 + P_f P_z} [\sin \gamma (P_x \cos 2\beta - P_z \sin 2\beta) - P_y \cos \beta \cos \gamma]. \end{aligned} \quad (16)$$

From these results, we see already the following qualitative conclusions:

(1) From Eqs. (8) and (12), we see that not only  $\rho_{00}^V(z, k_\perp | a, f)$  but also  $P/V$  depends on the polarization of the fragmenting quark  $q_f^0$  and that of the antiquark  $\bar{q}$  created in the fragmentation process. There is a simple relation between them, i.e.,

$$\rho_{00}^V(z, k_\perp | a, f) = (P/V) + \frac{2P_f \sin \beta}{3 + P_f P_z} (P_z \sin \beta - P_x \cos \beta). \quad (17)$$

We see that the relation is in general dependent on the momentum of the produced vector meson. If we take as usual  $k_\perp = 0.35 \text{ GeV}$ , we obtain  $\sin \beta = 2k_\perp / (z\sqrt{s}) \approx 0.0077/z$  for  $\sqrt{s} = 91 \text{ GeV}^{-2}$ , which is much less than 1 for large  $z$ . Hence we have approximately

$$\rho_{00}^V(z, k_\perp | a, f) \approx P/V. \quad (18)$$

For  $P_z = P_x = 0$ , we have  $\rho_{00}^V(z, k_\perp | a, f) = P/V = 1/3$ . This is the result that is expected<sup>17</sup> in the unpolarized case.

(2) From Eqs. (13-16), we see that the non-diagonal elements  $\rho_{1,-1}^V$  and  $(\rho_{1,0}^V - \rho_{0,-1}^V)$  depend not only on  $\beta$  but also on  $\gamma$  which is a function of  $\phi$  and other variables. Since for a given  $k_\perp$  and a given  $z$ ,  $\phi$  is distributed uniformly, we should average over  $\phi$  for these quantities. For  $k_\perp \ll (z\sqrt{s}/2)$ , we expand  $\cos \gamma$  and  $\sin \gamma$  in terms of  $k_\perp/k_z$  and keep only terms up to  $k_\perp/k_z$ , then average over  $\phi$ , we obtain  $\langle \cos \gamma \rangle \approx -\tan \beta \cot \theta/2$ ;  $\langle \sin \gamma \rangle \approx 0$ ;  $\langle \cos 2\gamma \rangle \approx -\tan^2 \beta$  and  $\langle \sin 2\gamma \rangle \approx 0$ . We insert them into Eqs. (13-16) and obtain

$$\text{Re} \rho_{1,-1}^V(z, k_\perp | a, f) \approx -\frac{P_f \sin \beta}{3 + P_f P_z} \tan^2 \beta (P_z \sin \beta - P_x \cos \beta), \quad (19)$$

$$\text{Im} \rho_{1,-1}^V(z, k_\perp | a, f) \approx -\frac{P_f \sin \beta}{3 + P_f P_z} P_y \tan^2 \beta, \quad (20)$$

$$\text{Re}[\rho_{1,0}^V(z, k_\perp | a, f) - \rho_{0,-1}^V(z, k_\perp | a, f)] \approx -\frac{\sqrt{2} P_f \tan \beta \cot \theta}{2(3 + P_f P_z)} (P_x \cos 2\beta - P_z \sin 2\beta), \quad (21)$$

$$\text{Im}[\rho_{1,0}^V(z, k_\perp | a, f) - \rho_{0,-1}^V(z, k_\perp | a, f)] \approx \frac{\sqrt{2} P_f P_y \sin \beta \cot \theta}{2(3 + P_f P_z)}. \quad (22)$$

We see that they contain at least one  $\sin \beta$  which is of the order of 0.01 at large  $z$ . This shows that these non-diagonal elements obtained in this way are all very small. More precisely, both  $\text{Re} \rho_{1,-1}$  and  $\text{Im} \rho_{1,-1}$  should be smaller than  $10^{-6}$  and  $\text{Re}[\rho_{1,0} - \rho_{0,-1}]$  and  $\text{Im}[\rho_{1,0} - \rho_{0,-1}]$  should be small than  $10^{-2}$ . Significant nonzero results of these elements may imply significant interferences<sup>18</sup> between the fragmentation of  $q_f^0$  and that of  $\bar{q}_f^0$ , which are

not taken into account here. Experimental results<sup>13–15</sup> from OPAL and DELPHI for  $Im\rho_{1,-1}$ ,  $Re[\rho_{1,0} - \rho_{0,-1}]$  and  $Im[\rho_{1,0} - \rho_{0,-1}]$  seem to show no deviation from the above-mentioned expectations, there is however signature of deviation for  $Re\rho_{1,-1}$ , but the statistics are still too low to make any definite conclusions.

(3) From Eq. (12), we see that if  $P_z = 0$ , i.e., the  $\bar{q}$  is unpolarized in the moving direction of  $q_f^0$ , we obtain that

$$\rho_{00}^V(z, k_\perp | a, f)|_{P_z=0} = (1 - P_f P_x \sin 2\beta)/3. \quad (23)$$

As we discussed above,  $\sin 2\beta \ll 1$  for large  $z$ , so that  $\rho_{00}^V(z, k_\perp | a, f)|_{P_z=0} \approx 1/3$ . This shows clearly that  $P_z \neq 0$  is a necessary condition for  $\rho_{00}^V \neq 1/3$  at large  $z$ . Furthermore, neglecting terms proportional to  $\sin \beta$  in Eq. (12), we obtain,

$$\rho_{00}^V(z, k_\perp | a, f) \approx (1 - P_f P_z)/(3 + P_f P_z). \quad (24)$$

This shows that  $\rho_{00}^V(z, k_\perp | a, f) > 1/3$  if  $P_f$  and  $P_z$  have opposite sign. Both OPAL and DELPHI data [13-16] explicitly show that  $\rho_{00}^V > 1/3$  for all the vector mesons except for  $\omega$ . This implies that  $P_z \neq 0$  and has the opposite sign as  $P_f$  in these cases.

After we obtain the results for  $\rho^V(z|a, f)$  and  $\rho^V(z|b)$ , we can calculate  $\rho^V(z)$  if we know the average numbers  $\langle n(z|a, f) \rangle$  and  $\langle n(z|b) \rangle$ . These average numbers are determined by the hadronization mechanism and should be independent of the polarization of the initial quarks. Hence, we can calculate them using a hadronization model which gives a good description of the unpolarized data for multiparticle production in high energy reactions. Presently, such calculation can only be carried out using a Monte-Carlo event generator. We use the LUND string fragmentation model<sup>19</sup> implemented by JETSET<sup>20</sup> to do this calculation.

Using the event generator JETSET, we calculated  $\langle n(z|a, f) \rangle$  and  $\langle n(z|b) \rangle$ . We use Eq. (24) to calculate  $\rho_{00}^V(z, k_\perp | a, f)$  in which the only free parameter is  $P_z$ . We first calculate  $\rho_{00}$  for  $K^{*0}$  as a function of  $z$  since the corresponding data is available<sup>14</sup>. We found out that, to fit the data, the value of  $P_z$  has to be quite large. In Fig. 2 we show the results obtained by taking  $P_z = 0.48$ . We see that the data can reasonably well be fitted.



Similar calculations can certainly be made for other vector mesons such as  $\rho$ ,  $\phi$ ,  $D^{*\pm}$ ,  $B^*$  etc. But, for these mesons, only the average values of  $\rho_{00}$  in certain  $z$  ranges are available. It can easily be obtained from Eq. (1), such average values are given by,

$$\langle \rho_{00}^V \rangle = \sum_f \frac{\langle n^V(a, f) \rangle}{\langle n^V \rangle} \rho_{00}^V(a, f) + \frac{\langle n^V(b) \rangle}{\langle n^V \rangle} \rho_{00}^V(b), \quad (25)$$

where  $\rho_{00}^V(a, f)$  is given by Eq. (24) and  $\rho_{00}^V(b) = 1/3$ ;  $\langle n^V(a, f) \rangle$  and  $\langle n^V(b) \rangle$  are the number of vector mesons from (a) and (b) in the corresponding  $z$  range respectively and  $\langle n^V \rangle$  the total number of vector meson in that  $z$  range. Using JETSET, we calculate  $\langle n^V(a, f) \rangle$  and  $\langle n^V(b) \rangle$ . After that we can determine the  $P_z$  which we need to fit the data on  $\langle \rho_{00}^V \rangle$ . We found out that, for most of the vector mesons, the resulting  $P_z$  can be written as  $P_z = -\alpha P_f$  with  $\alpha$  is common for most of the  $q_f^0$ 's. In Table 1, we show the obtained results by choosing  $\alpha = 0.51$  as determined above from the data for  $K^{*0}$ . We see that the data can be fitted reasonably well except those for  $\omega$  and  $B^*$ . The data of  $\langle \rho_{00}^\omega \rangle$  can only be fitted by taking  $\alpha$  as negative. The deviation in the case of  $B^*$  may be attributed to the helicity flip of  $b$ -quark caused by gluon radiation, which is negligible for light quarks.

The results except that for  $\omega$  can be understood qualitatively in the string fragmentation model. Here, it was shown that<sup>19</sup> the probability to creat a meson of mass  $m$  is proportional to  $e^{-bm^2/x}$ , where  $x$  is the fractional energy of meson from the fragmenting quark and  $b$  is a positive parameter in LUND model. For a polarized quark  $q_f^0$  with polarization  $P_f$ , if the spin of  $\bar{q}$  is in the opposite direction as that of  $q_f^0$ , the resulting mesons can be a vector meson or a pseudoscalar meson with equal probabilities. The average mass is  $m_1 = (m_V + m_P)/2$ , where  $m_V$  and  $m_P$  are the masses for the vector meson and pseudoscalar meson respectively. If the spin of  $\bar{q}$  is in the same direction as that of  $q_f^0$ , only vector meson can be created and its mass is  $m_2 = m_V$ . It is clear that  $m_1^2 < m_2^2$ , thus the corresponding probability is large for the former case than that for the latter. This leads to a longitudinal polarization of  $\bar{q}$  and the polarization is proportional to  $P_f$ , i.e,  $P_z = -\alpha_{str}^f P_f$ ,

$$\alpha_{str}^f = \frac{2}{e^{-b(m_2^2 - m_1^2)/x} + 1} - 1 \quad (26)$$

We can see that the sign of  $P_z$  is indeed opposite to that of  $P_f$ . In case of  $K^{*0}$  which contains an initial  $d$  or  $\bar{s}$  we obtain  $\alpha_{str}^f = 0.3$  for  $x = 0.3$  by taking  $b = 0.58 GeV^{-2}$  as was used in JETSET<sup>20</sup>.

In summary, we calculated the spin density matrix of the vector meson produced in  $e^+e^-$  annihilation at  $Z^0$  pole from a direct sum of the spin of the polarized fragmenting quark and that of the antiquark created in the fragmentation process. The result for  $\rho_{00}$  implies a significant polarization for the antiquark which is created in the fragmentation process and combines with the fragmenting quark to form the vector meson in the opposite direction as that of the fragmenting quark. The polarization can approximately be written as  $P_z = -\alpha P_f$  and  $\alpha \approx 0.5$  for most of the mesons.

We thank Li Shi-yuan, Xie Qu-bing and other members of the theoretical particle physics in Shandong for helpful discussions. This work was supported in part by the National Science Foundation of China (NSFC) and the Education Ministry of China.

## REFERENCES

1. ALEPH Collaboration, D. Buskulic et al., Phys. Lett. **B374**, 319 (1996);
2. OPAL Collaboration, K. Ackerstaff et al., Eur. Phys. J. **C2**, 49 (1998).
3. J.E. Augustin and F.M. Renard, Nucl. Phys. **B162**, 341 (1980).
4. R.L. Jaffe and Ji Xiang-dong, Phys. Rev. Lett. **67**, 552 (1991); Nucl. Phys. **B375**, 527 (1992).
5. M. Burkardt and R.L. Jaffe, Phys. Rev. Lett. **70**, 2537 (1993).
6. G. Gustafson and J. Häkkinen, Phys. Lett. **B303**, 350 (1993).
7. R.L. Jaffe, Phys. Rev. **D54**, R6581 (1996).
8. C. Boros and Liang Zuo-tang, Phys. Rev. **D57**, 4491 (1998).
9. A. Kotzinian, A. Bravar and D. von Harrach, Eur. Phys. J. **C2**, 329 (1998).
10. B.Q. Ma, I. Schmidt and J.J. Yang, Phys. Rev. **D61**, 034017 (2000); **62**, 114009 (2000); Phys. Lett. **B489**, 293 (2000).
11. Liu Chun-xiu and Liang Zuo-tang, Phys. Rev. **D62**, 094001 (2000).
12. DELPHI Collaboration, P. Abreu et al., Z. Phys. **C68**, 353 (1995);  
ALEPH Collaboration, D. Buskulic et al., Z. Phys. **C69**, 393 (1995).
13. DELPHI Collaboration, P. Abreu et al., Phys. Lett. **B406**, 271(1997).
14. OPAL Collaboration, K. Ackerstaff et al., Phys. Lett. **B412**, 210 (1997).
15. OPAL Collaboration, K. Ackerstaff et al., Z. Phys. **C74**, 437 (1997).
16. OPAL Collaboration, G. Abbiendi et al., Eur. Phys. J. **C16**, 61 (2000).
17. J.F. Donoghue, Phys. Rev. **D19**, 2806 (1979); I.I. Y. Bigi, Nuovo Cimento **A41**, 581 (1977).

18. M.A. Anselmino, M. Bertini, F. Murgia and P. Quintairos, Eur. Phys. J. **C2**, 539 (1998) and references therein.
19. B. Andersson, G. Gustafson, G. Ingelman and T.Sjöstrand, Phys. Rep. **97**, 31 (1983).
20. T. Sjöstrand, Comp. Phys. Commun. **39**, 347 (1986).

# TABLES

Table 1. Spin density matrix element  $\rho_{00}$  for different vector mesons obtained from Eq.(25) using  $P_z = -0.51P_f$ . The data are taken from Refs.[12-16].

meson	$\rho_{00}$	data	$z$ range
$\rho^\pm$	0.398	$0.373 \pm 0.052(\text{OPAL})$	$0.3 < z < 0.6$
$\rho^0$	0.428	$0.43 \pm 0.05(\text{DELPHI})$	$z > 0.4$
$\omega$	0.405	$0.142 \pm 0.114(\text{OPAL})$	$0.3 < z < 0.6$
$K^{*0}$	0.504	$0.46 \pm 0.08(\text{DELPHI})$	$z > 0.4$
$\phi$	0.557	$0.54 \pm 0.06 \pm 0.05(\text{OPAL})$	$z > 0.7$
		$0.55 \pm 0.10(\text{DELPHI})$	
$D^{*\pm}$	0.415	$0.40 \pm 0.02 \pm 0.01(\text{OPAL})$	$z > 0.5$
		$0.32 \pm 0.04 \pm 0.03(\text{DELPHI})$	
$B^*$	0.567	$0.33 \pm 0.06 \pm 0.05(\text{ALEPH})$	$0 < z < 1$
		$0.36 \pm 0.06 \pm 0.07(\text{OPAL})$	

# FIGURES

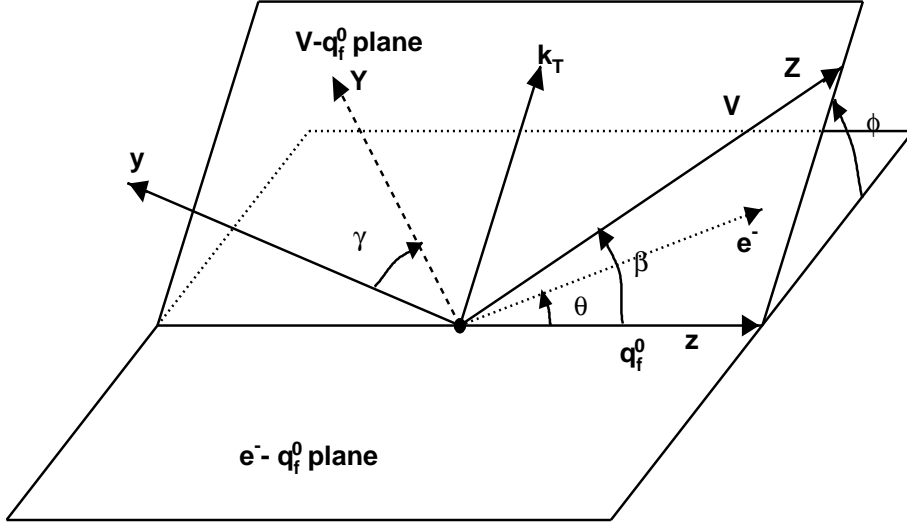


Fig. 1. Illustrating graph showing the relation between the helicity frame  $oxyz$  of the initial quark  $q_f^0$  and the helicity beam frame  $OXYZ$  of the produced vector meson  $V$ . There are three planes involved here: the horizontal plane determined by the incoming  $e^+e^-$  and outgoing  $q_f^0\bar{q}_f^0$ , the  $x-z$  plane, i.e.,  $V-q_f^0$  plane determined by the moving direction of  $q_f^0$  and that of  $V$  and  $\vec{e}_y = \vec{e}_z \times \vec{e}_{k_\perp} / |\vec{e}_z \times \vec{e}_{k_\perp}|$ , the  $X-Z$  plane determined by the moving direction of  $q_f^0$  and that of the  $e^-$  beam and  $\vec{e}_Y = \vec{e}_Z \times \vec{e}_{p_{e^-}} / |\vec{e}_Z \times \vec{e}_{p_{e^-}}|$ .

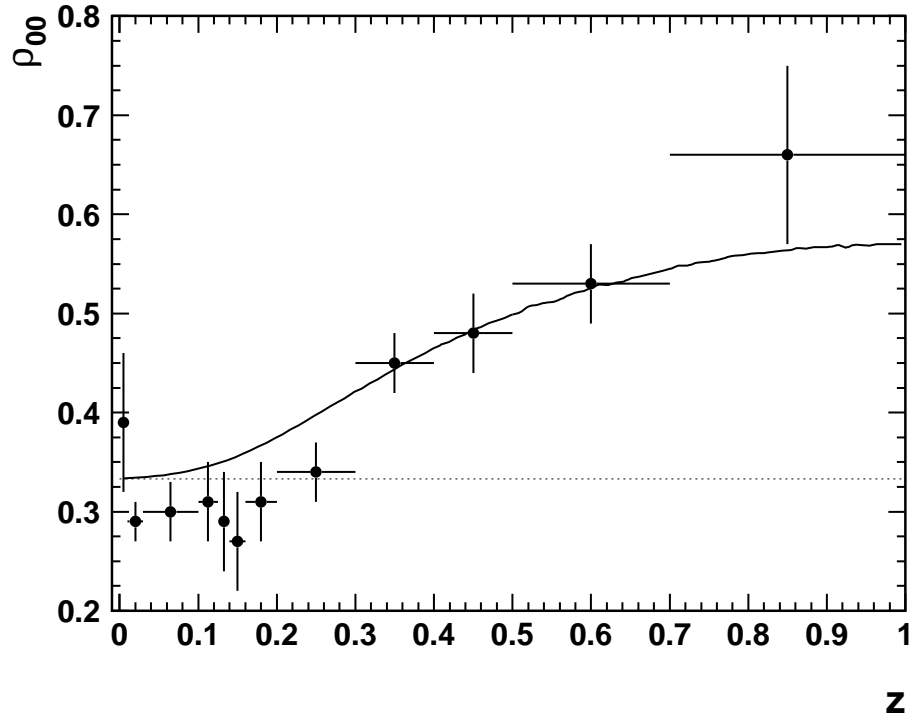


Fig. 2. Spin density matrix element  $\rho_{00}$  of  $K^{*0}$  produced in  $Z^0$  decay obtained from Eq. (24) by taking  $P_z = 0.48$  as a function of the momentum fraction  $z$ . The dotted straight line corresponds to the unpolarized case ( $1/3$ ); the data are from Ref. [16].

Figure caption:

Fig. 1: Illustrating graph showing the relation between the helicity frame  $oxyz$  of the initial quark  $q_f^0$  and the helicity beam frame  $OXYZ$  of the produced vector meson  $V$ . There are three planes involved here: the horizontal plane determined by the incoming  $e^+e^-$  and outgoing  $q_f^0\bar{q}_f^0$ , the  $x-z$  plane, i.e.,  $V-q_f^0$  plane determined by the moving direction of  $q_f^0$  and that of  $V$  and  $\vec{e}_y = \vec{e}_z \times \vec{e}_{k_\perp}/|\vec{e}_z \times \vec{e}_{k_\perp}|$ , the  $X-Z$  plane determined by the moving direction of  $q_f^0$  and that of the  $e^-$  beam and  $\vec{e}_Y = \vec{e}_Z \times \vec{e}_{p_{e^-}}/|\vec{e}_Z \times \vec{e}_{p_{e^-}}|$ .

Fig. 2: Spin density matrix element  $\rho_{00}$  of  $K^{*0}$  produced in  $Z^0$  decay obtained from Eq. (24) by taking  $P_z = 0.48$  as a function of the momentum fraction  $z$ . The dotted straight line corresponds to the unpolarized case ( $1/3$ ); the data are from Ref. [16].

Table caption:

Tab. 1: Spin density matrix element  $\rho_{00}$  for different vector mesons obtained from Eq.(25) using  $P_z = -0.51P_f$ . The data are taken from Refs.[12-16].